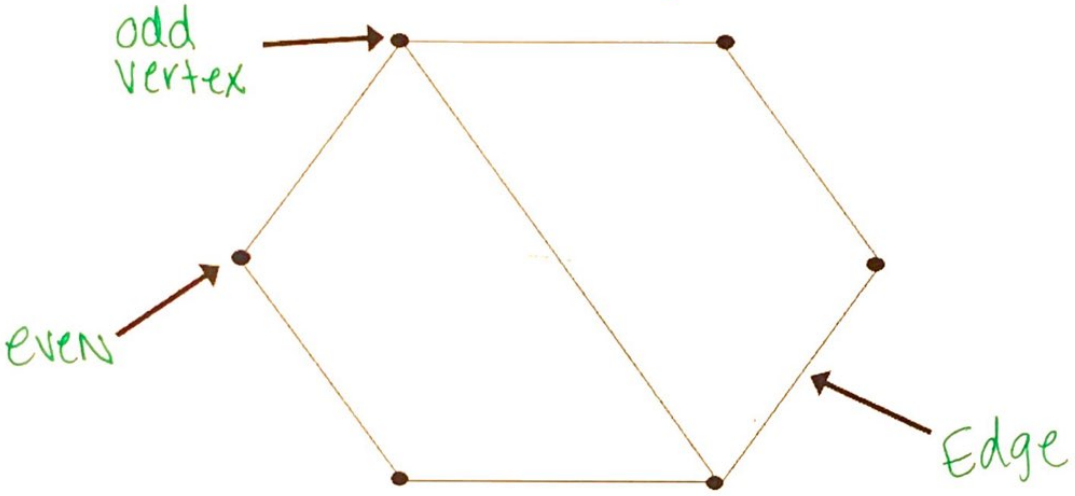
HILLGROVE
HIGH SCHOOL

AMDM: Unit 3 – Networks
3.1 – Euler Paths & Circuits – February 11

- A ~~graph~~ network is a figure made up of points (vertices) connected by non-intersecting edges.
- A vertex is the intersection of two edges.
- A vertex is odd if it is connected to an odd number of edges
- A vertex is even if it is connected to an even number of edges



Euler path - a continuous path that passes through every edge once and only once.

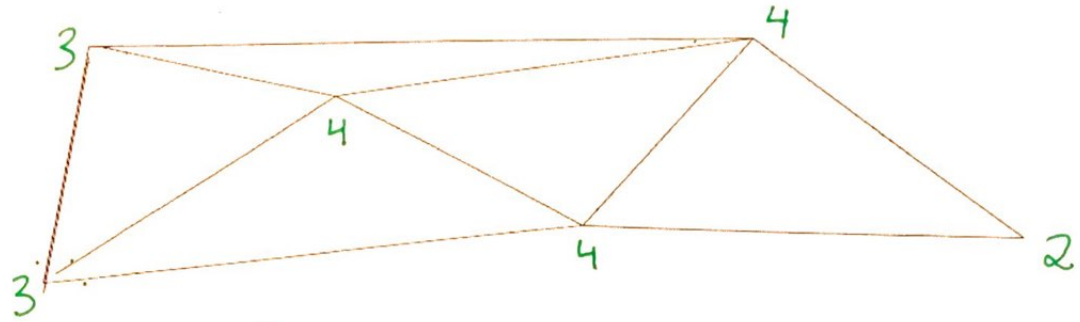
Euler circuit - when an Euler path begins and ends at the same vertex.

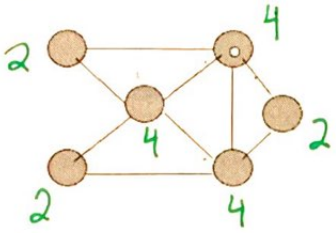
Euler's 1st Theorem

If a graph has any vertices of odd degree, then it cannot have an Euler circuit.

If a graph is connected and every vertex has an even degree, then it has at least one Euler circuit (usually more).

Is This An Euler Path or Circuit?





1) How many vertices are there? 6

2) How many edges are there? 9

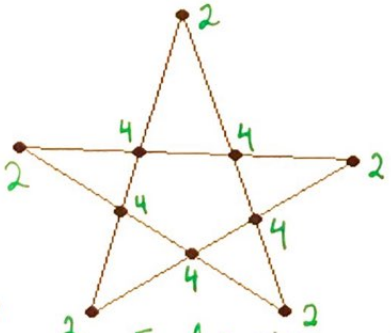
3)How many vertices have a degree of 2? 3

4) How many vertices have a degree of 4? 3

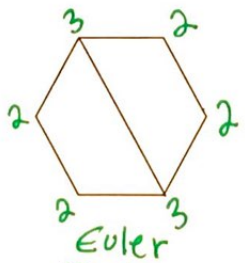
Draw a Euler circuit starting at the vertex with a white dot.

Remember: A circuit travels along every ~~path~~ edge exactly once and may pass through vertices multiple times before ending at the starting vertex.

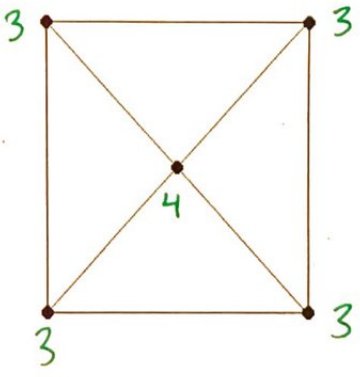
Can you find an Euler Path or Circuit for the following networks?



Euler Circuit



Euler Path



Conclusions: Based on the observations of your table:

A graph with all vertices being even contains an Euler Circuit.

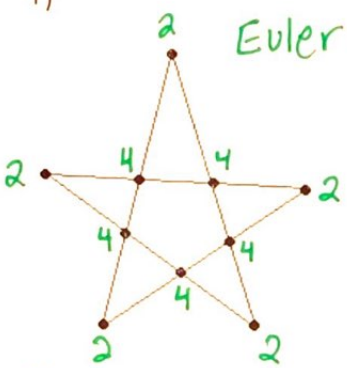
A graph with 2 odd vertices and the rest even vertices contains an Euler path.

A graph with more than two odd vertices does not contain an Euler path or circuit

3.2 Practice – Euler Circuits **Unit 3: Networks and Graphs** **Name** Key

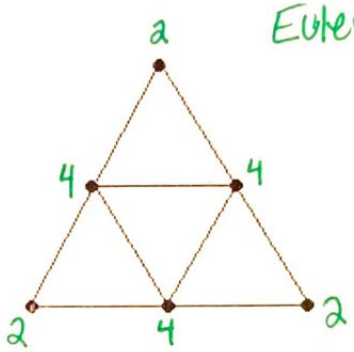
Directions: Use the table at the bottom of the page to determine if the following graphs contain a Euler Circuit, Euler Path, or Neither:

1)



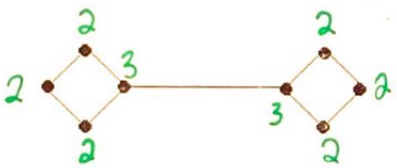
Euler Circuit

2)



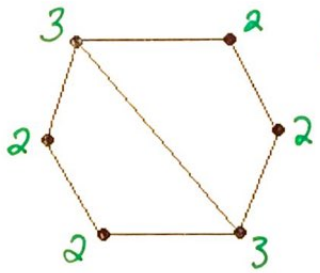
Euler Circuit

3)



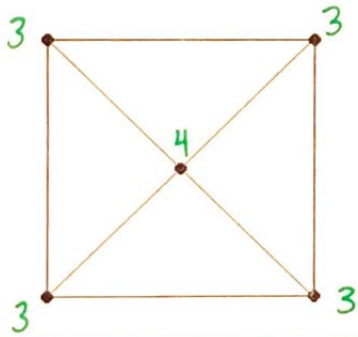
Euler Path

4)



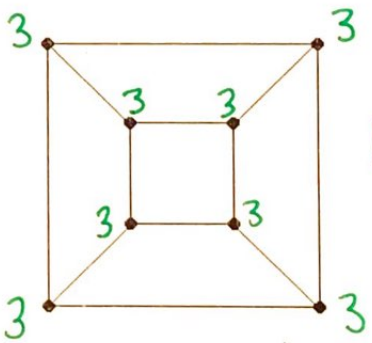
Euler Path

5)



Neither

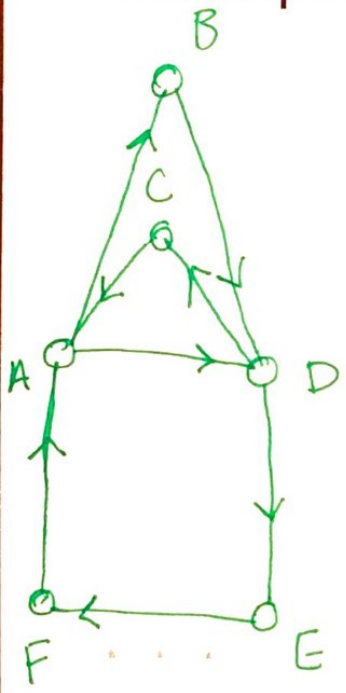
6)

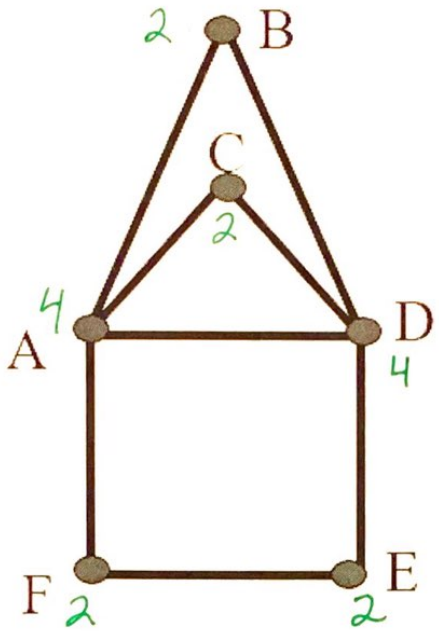


Neither

Graph	Number of odd vertices (vertices connected to an odd number of edges)	Number of even vertices (vertices connected to an even number of edges)	What does the path contain? Euler path = P Euler circuit = C Neither = N
1	0	10	C
2	0	6	C
3	2	6	P
4	2	4	P
5	4	1	N
6	8	0	N

Example 1: Name a Euler circuit





One possible solution is

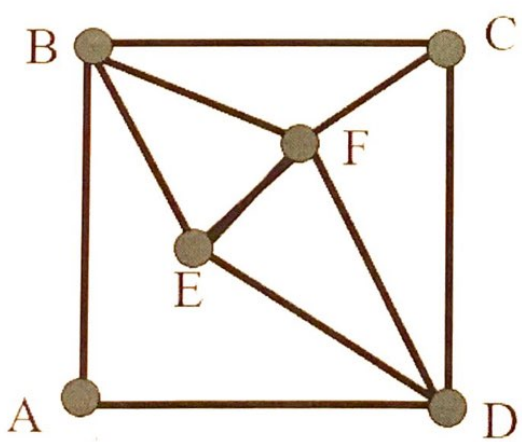
D,E,F,A,D,C,A,B,D

b) Can you find another one?

A B D C A D E F A

Given A,B,E,F,B,C,D,F,E,D is this a Euler path or circuit or neither?

How can you tell? Explain your answer



Find a Euler circuit if possible, if not list a Euler path

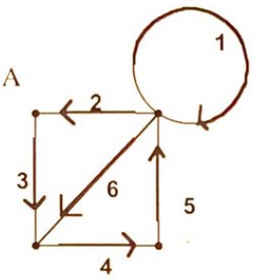
2 odd, so Euler path starting at E or C.

E B A D E F D C B F C

Euler circuit and path worksheet:

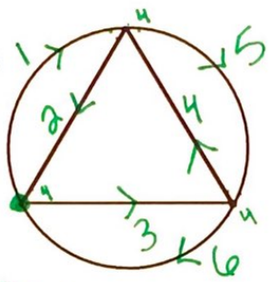
Part 1: For each of these vertex-edge graphs, try to trace it (without lifting your pen from the paper, and without tracing any edge twice). If you succeed, number the edges in the order you used them (puting on arrows is optional), and circle whether you found an Euler circuit or an Euler path. The first one is done for you

A



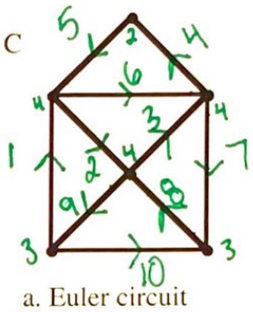
a. Euler circuit
b. Euler path
c. Not traceable

B



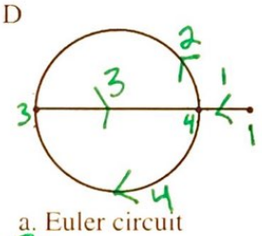
a. Euler circuit
b. Euler path
c. Not traceable

C



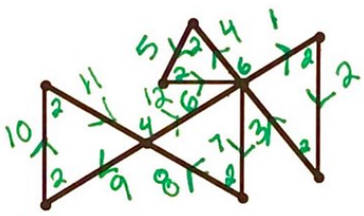
a. Euler circuit
b. Euler path
c. Not traceable

D



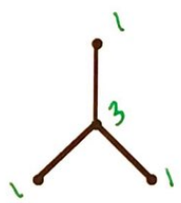
a. Euler circuit
b. Euler path
c. Not traceable

E



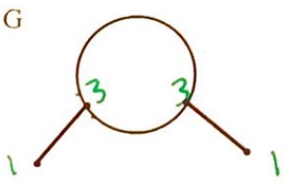
a. Euler circuit
b. Euler path
c. Not traceable

F



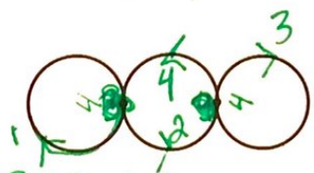
a. Euler circuit
b. Euler path
c. Not traceable

G



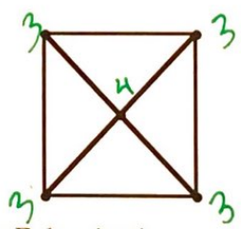
a. Euler circuit
b. Euler path
c. Not traceable

H



a. Euler circuit
b. Euler path
c. Not traceable

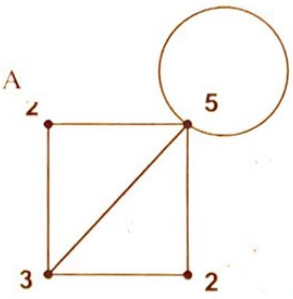
I



a. Euler circuit
b. Euler path
c. Not traceable

Part 2: For each of these, write the valence number next to each vertex, then tell how many vertices are odd (odd valence #), and how many are even (the valence # is even)

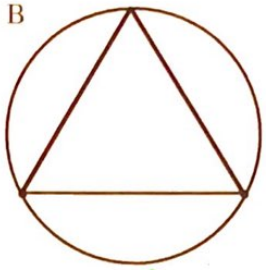
A



odd V: 2

even V: 2

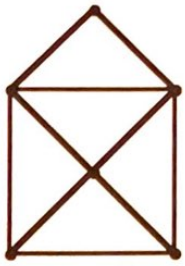
B



odd V: 0

even V: 3

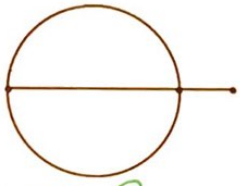
C



odd V: 2

even V: 4

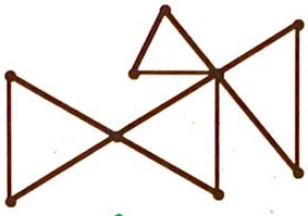
D



odd V: 2

even V: 1

E



odd V: 0

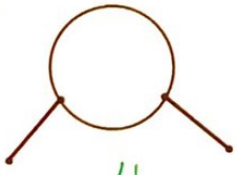
even V: 9

F

odd V: 4

even V: 0

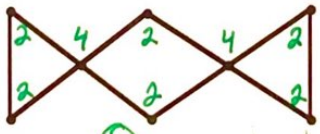
G



odd V: 4

even V: 0

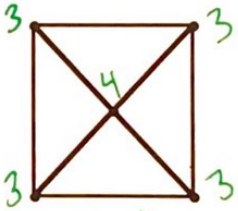
H



odd V: 0

even V: 8

I



odd V: 4

even V: 1

Put it together:

3 of the graphs have Euler circuits. How many odd vertices do they have? 0

3 of the graphs have Euler paths. How many odd vertices do they have? 2

3 of the graphs are not traceable. How many odd vertices do they have? > 2