Example 1: What Is a Fair Game?

A set of paper bags are labeled A, B, C, D, E, F. The game is played by choosing two bags. Each of five of the bags holds $1.00, and one of the bags holds $10.00. The total of the amounts on the two bags chosen is the amount of winnings for the participant.

Exercises 1–5

1. What are the possible total amounts of money you could win if you choose two bags?

Two $1 bags ⇒ $\boxed{\$2 \text{ won}}$

One $1 bag + One $10 bag ⇒ $\boxed{\$11 \text{ won}}$

2. If you pick two bags at random:

   a. How likely is it that you win $2.00?

$$\frac{5}{6} \cdot \frac{4}{5} = \frac{20}{30} = \boxed{\frac{2}{3}}$$

   b. How likely is it that you win $11.00?

$$\frac{1}{6} \cdot \frac{5}{5} \text{ OR } (+) \frac{5}{6} \cdot \frac{1}{5} = \frac{5}{30} + \frac{5}{30} = \boxed{\frac{1}{3}}$$

3. Based on Exercise 2, how much should you expect to win on average per game if you played this game many times?

$$2\left(\frac{2}{3}\right) + 11\left(\frac{1}{3}\right) = \$5$$

4. To play the game, you must pay to play. The price of the game is set so that the game is fair. What do you think is meant by a fair game in the context of playing this game?

5. How much should you be willing to pay for a game if the game is to be a fair one? Explain.

Example 2: Deciding Between Two Alternatives

You have a chore to do around the house for which your mom plans to pay you $10.00. When you are done, your mom, being a mathematics teacher, gives you the opportunity to change the amount that you are paid by playing a game.

She puts three $2.00 bills in a bag along with two $5.00 bills and one $20.00 bill. She says that you can take the $10.00 she offered originally or you can play the game by reaching into the bag and selecting two bills without looking. You get to keep these two bills as your payment

Exercises 6 $2 $2 $2 $5 $5 $20

6. Do you think you should take your mom's original payment of $10.00 or play the "bag" game? In other words, is this game a fair alternative to getting paid $10.00?

outcomes: $\$4 \rightarrow P(\$4) = \frac{3}{6} \cdot \frac{2}{5} = \frac{6}{30} = \frac{1}{5}$

$\$7 \rightarrow P(\$7) = \left(\frac{3}{6} \cdot \frac{2}{5}\right) + \left(\frac{2}{6} \cdot \frac{3}{5}\right) = \frac{2}{5}$

$\$10 \rightarrow P(\$10) = \left(\frac{2}{6} \cdot \frac{1}{5}\right) = \frac{2}{30} = \frac{1}{15}$

$\$22 \rightarrow P(\$22) = \left(\frac{3}{6} \cdot \frac{1}{5}\right) + \left(\frac{1}{6} \cdot \frac{3}{5}\right) = \frac{1}{5}$

$\$25 \rightarrow P(\$25) = \left(\frac{2}{6} \cdot \frac{1}{5}\right) + \left(\frac{1}{6} \cdot \frac{2}{5}\right) = \frac{2}{15}$

Expected Value $= 4(1/5) + 7\left(\frac{2}{5}\right) + 10\left(\frac{1}{15}\right) + 22\left(\frac{1}{5}\right) + 25\left(\frac{2}{15}\right)$

$= \$12.00$

– The game is in your favor because $\$12 > \$10$

Exercises 7

7. Alter the contents of the bag in Example 2 to create a game that would be a fair alternative to getting paid $10.00. You must keep six bills in the bag, but you can choose to include bill-sized pieces of paper that are marked as $0.00 to represent a $0.0 bill.